Coherence
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Abstract
Quite often, reality arranges coherence in a standard way. This way produces recognizable phenomena that occur in all places where reality controls coherence in that way. The document shows the relation between the hopping path cycle of elementary particles and the Lagrangian that describes their kinematic behavior. Further, the paper describes how stochastic processes control the binding of stochastically controlled objects.

Introduction
This paper focusses on distributions of similar discrete objects. First, we consider distributions that are not composed of other distributions. The distributions describe the probability of detecting an object at a spatial location. A stochastic process is supposed to distribute the objects over these locations. The process owns a characteristic function, which ensures that the process generates a coherent detection location swarm. The fact that the characteristic function equals the Fourier transform of the location detection probability distribution ensures the coherence of the distribution. The variance of the location density distribution and the variance of the characteristic function are coupled.

In general, if \( f \) is an arbitrary probability density distribution and \( F \) is its Fourier transform, then

\[
\text{var}(f) \text{var}(F) \geq 1
\]

This holds for one, two, or three-dimensional distributions. The equal sign holds for normal distributions. You might recognize the role of the uncertainty principle in this formula.

The existence of the Fourier transform ensures that the detection location swarm behaves as a wave package. Usually moving wave packages disperse, but here the stochastic process recurrently regenerates the wave package. Thus, even though the objects themselves are discrete objects, the swarm may show interference phenomena. If the characteristic function is the Fourier transform of a pattern, then the generated swarm behaves as that pattern.

It is well-known that for passing distributions of tiny objects small apertures behave as Fourier operators. If the characteristic function is described by a pair of slits, then the location probability distribution behaves as the Fourier transform of this pattern. Camera obscura is another example.

Fourier optics
A halo around a galaxy, a glass lens, a specially shaped gravitation potential, a tiny aperture or a set of tiny apertures can all figure as imaging devices. What they have in common is that they curve the path of the objects that form the image. The image is a surface. That surface need not be flat. It is also possible that the image is a set of focus regions. In a linear imaging process, the image is a blurred copy of the source object. Each source location corresponds with a single target location. The image of a point-like source is the point spread function. The point spread function describes the distribution of the point locations that form the image of a point-like source. A stochastic process controls the generated target locations. The characteristic function of this stochastic process equals
the optical transfer function of the imaging process. Usually, the optical transfer function depends on a series of circumstances. It depends on the angular distribution of the participating point source objects. It usually depends on the energy of the participating point source objects. It also depends on the phase coherence of the point source objects. These distributions may change with the selected source point. Often the imaging devices form an imaging chain. In that case, the target of a previous member of the chain is the source of the next member of the chain. By reckoning the influences of the mentioned distributions, the optical transfer function of the linear chain can be computed from the product of the optical transfer functions of the members. The linear operation means that a superposition of objects produces a corresponding superposition of images.

This means that the imaging chain is described by a stochastic process, which owns a characteristic function that equals the product of the characteristic functions of the members of the chain.

**Elementary particles**

Elementary particles reside on a private platform, which is a separable quaternionic Hilbert space. This Hilbert space applies a selected version of the quaternionic number system, and this version determines the symmetry properties of the platform. The elementary particle inherits these symmetry properties. A private stochastic process that owns a characteristic function generates the landing locations of the hopping path of the particle. This results in the recurrent regeneration of a coherent hop landing location swarm. The location density distribution of this swarm equals the squared modulus of the wavefunction of the particle. It also equals the Fourier transform of the characteristic function of the process.

The hopping path appears to be nearly cyclic. After a full cycle, it returns to nearly the same location. This means that the geometric center of the swarm moves with the resulting difference. Thus, where the particle hops violently, the swarm moves rather smoothly. Its movement appears to be controlled by a displacement generator, which locates in Fourier space. This situation invites to consider the hopping path as a chain of imaging devices that try to generate the location of the geometric center of the swarm as the final image. The spatial spectrum of the hop acts as the optical transfer function of the hop. To investigate this setup, we split the hop into three steps. The first step represents a step from the eigenvector that represents the current location $\vec{a}$ in configuration space to the eigenvector that represents the displacement generator $\vec{p}$ in Fourier space. The effect is contained in the inner product $\langle \vec{a} | \vec{p} \rangle$ of these vectors. The second step concerns the action of the displacement generator $\exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle \right)$. The third step is the step back from $\vec{p}$ in Fourier space to location $\vec{a}_{i+1}$ in configuration space. The effect is contained in the inner product $\langle \vec{p} | \vec{a}_{i+1} \rangle$ of these vectors.

Together these steps represent the factor $\langle \vec{a} | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle \right) \langle \vec{p} | \vec{a}_{i+1} \rangle$.

The next hop is represented by $\langle \vec{a}_{i+1} | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_{i+1} \rangle \right) \langle \vec{p} | \vec{a}_{i+2} \rangle$.

Combined these terms deliver

$$\langle \vec{a} | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+1} - \vec{a}_i \rangle \right) \langle \vec{p} | \vec{a}_{i+1} \rangle \langle \vec{a}_{i+1} | \vec{p} \rangle \exp\left(\langle \vec{p}, \vec{a}_{i+2} - \vec{a}_{i+1} \rangle \right) \langle \vec{p} | \vec{a}_{i+2} \rangle$$
For the complete hopping path cycle that contains $N$ terms, this gives

$$
\langle \hat{a}_i | \hat{p} \rangle \exp \left(\sum_{i=1}^{N} \langle \hat{p}, \hat{a}_{i+1} - \hat{a}_i \rangle \right) \langle \hat{p} | \hat{a}_N \rangle
$$

$$
= \langle \hat{a}_i | \hat{p} \rangle \exp \left(\langle \hat{p}, \hat{a}_{i+2} - \hat{a}_i \rangle \right) \langle \hat{p} | \hat{a}_{i+2} \rangle
$$

This invites the following approximation:

$$
\int L d\tau = \sum_{i=1}^{N} \langle \hat{p}, \hat{a}_{i+1} - \hat{a}_i \rangle
$$

$L = \langle \hat{p}, \hat{q} \rangle$

Here $L$ is the Lagrangian and $\hat{q}$ is the location of the geometric center of the swarm.

This definition results in the following equations of motion

$$
\frac{\partial L}{\partial \dot{q}_i} = \dot{p}_i
$$

$$
\frac{\partial L}{\partial \dot{q}_i} = p_i
$$

$$
\frac{d}{d\tau} \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}
$$

Thus, if the hopping path cycle is considered as an imaging chain, then its focusing produces the same effect as the Lagrangian does.

This algorithm does the inverse of what the well-known path algorithm does. That algorithm starts with the Lagrangian and integrates over all possible paths. Here we only apply a hopping path cycle, and we end with the Lagrangian.

**Composed distributions**

Elementary particles are elementary modules. Together, they constitute all other modules that exist in the universe. Some of the modules constitute modular systems.

Elementary modules are controlled by private elementary stochastic processes that own a characteristic function. These elementary modules reside on a floating platform that is represented by a separable quaternionic Hilbert space. A selected version of the quaternionic number system determines the symmetry properties of the platform.

Composed modules are also controlled by a private stochastic process that owns a characteristic function. When compared to the elementary stochastic processes, this stochastic process has a different nature. Its characteristic function equals a dynamic superposition of the characteristic functions of the components. These components are elementary modules, or they are themselves
composed modules. Due to this arrangement, the superposition coefficients can act as displacement generators that determine the internal locations of the components. Since the composed module moves as a single unit, the internal locations of the components oscillate. If the components are fermions, then these components must occupy different oscillation modes.

**Binding**

The oscillations bind the components. Effective binding of components appears to require that it is supported by the deformation of the embedding field. Deformation only occurs with the help of spherical pulse responses. These can only be generated by isotropic pulses. Electrons and positrons naturally generate isotropic pulses when their hops land onto the embedding field. Quarks must first conglomerate into colorless hadrons before the result can cause isotropic pulses. This phenomenon is known as color confinement.

**Compound modules**

Elementary particles reside on a private separable quaternionic Hilbert space. For a free elementary module, this Hilbert space floats with its selected version of the quaternionic number system over a background parameter space that is spanned by another version of the quaternionic number system. In a compound module, the elementary modules share the same geometric center. Thus, they no longer float with respect to each other. The target centers of the private stochastic processes of the components can still oscillate around this common geometric center. In fact, the fermionic components must apply different oscillation modes. Not the generated swarms oscillate. Only the target centers of the private stochastic processes oscillate.

In the compound modules, the symmetry-related properties of the components compensate each other as far as is possible. A neutral compound module has all its charges neutralized.

Physicists and chemists call compound modules atoms or atomic ions.

**Building blocks**

In the nineteenth century, atoms were considered to form the fundamental building block of matter. This changed in the first part of the twentieth century into the idea that the elementary particles take that role. The Hilbert book model changes this idea again by stating that shock fronts form the basics constituents of all other objects that occur in the universe. Spherical shock fronts constitute elementary particles and strings of equidistant one-dimensional shock fronts that obey \( E = h\nu \) implement the functionality of photons.

**References**

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